QUIZ 18 SOLUTIONS: LESSON 22
MARCH 18, 2019

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. Write as much work as you need to demonstrate to me that you understand the concepts involved. If you have any questions, raise your hand and I will come over to you.

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

1. [5 pts] Use the chain rule to find $\frac{dz}{dt}$ when $t = 1$:

$$z = \frac{6x}{y}, \quad x = 6e^{t-1}, \quad y = 3t^2$$

$\frac{\partial z}{\partial x} = \frac{\partial}{\partial x}\left(\frac{6x}{y}\right) = \frac{6}{y}\left[\frac{\partial}{\partial x}(x)\right] = \frac{6}{y}$

$\frac{\partial z}{\partial y} = \frac{\partial}{\partial y}\left(\frac{6x}{y}\right) = 6x\left[\frac{\partial}{\partial y}\left(\frac{1}{y}\right)\right] = 6x\left[\frac{-1}{y^2}\right] = \frac{-6x}{y^2}$

$\frac{dx}{dt} = \frac{d}{dt}(6e^{t-1}) = 6e^{t-1}, \quad \frac{dy}{dt} = \frac{d}{dt}(3t^2) = 6t$

When $t=1$, $x(1) = 6e^{1-1} = 6e^0 = 6$

$y(1) = 3(1)^2 = 3$

$\frac{dz}{dt} = \frac{6}{y}\left[6e^{t-1}\right] + \left(\frac{-6x}{y^2}\right)[6t]$

$\frac{dz}{dt}(t=1) = \frac{6}{3}\left[6e^{1-1}\right] - \frac{6(6)}{(3)^2}[6(1)]$

($e^{1-1} = e^0 = 1$)

$= \frac{36}{3} - \frac{216}{9} = 12 - 24 = \boxed{-12}$

2. [5 pts] Consider a right circular cylinder (Vol $= \pi r^2 h$). Suppose

- the radius is increasing at a rate of 6 inches per minute and
- the height is decreasing at a rate of 13 inches per minute.

Find rate of change of the **volume** when

- the radius is 16 inches and
- the height is 31 inches.

Round your answer to 3 decimal places.

$$\frac{dV}{dt} = \frac{\partial V}{\partial r}\frac{dr}{dt} + \frac{\partial V}{\partial h}\frac{dh}{dt}$$

$$\frac{\partial V}{\partial r} = \frac{\partial}{\partial r}(\pi r^2 h) = 2\pi rh, \quad \frac{\partial V}{\partial h} = \frac{\partial}{\partial h}(\pi r^2 h) = \pi r^2$$

$$\frac{dr}{dt} = +6, \quad \frac{dh}{dt} = -13$$

$$r = 16, \quad h = 31$$

$$\frac{dV}{dt} = [2\pi rh](6) + [\pi r^2](-13)$$

$$= [2\pi(16)(31)](6) + [\pi(16)^2](-13)$$

$$= 2624\pi$$

$$\approx \boxed{8243.539}$$